

Cultivating Complex Analysis: Meromorphic functions (5.2.4)

Jiří Lebl

Departemento pri Matematiko de Oklahoma Ŝtata Universitato

Definition

A holomorphic function $f: U \setminus S \rightarrow \mathbb{C}$ with poles on a discrete set $S \subset U$ is said to be *meromorphic*.

Definition

A holomorphic function $f: U \setminus S \rightarrow \mathbb{C}$ with poles on a discrete set $S \subset U$ is said to be *meromorphic*.

If $p \in S$ is a pole, set $f(p) = \infty$ to get a function

$$f: U \rightarrow \mathbb{C}_\infty.$$

Definition

A holomorphic function $f: U \setminus S \rightarrow \mathbb{C}$ with poles on a discrete set $S \subset U$ is said to be *meromorphic*.

If $p \in S$ is a pole, set $f(p) = \infty$ to get a function

$$f: U \rightarrow \mathbb{C}_\infty.$$

The extended f is continuous.

Definition

A holomorphic function $f: U \setminus S \rightarrow \mathbb{C}$ with poles on a discrete set $S \subset U$ is said to be *meromorphic*.

If $p \in S$ is a pole, set $f(p) = \infty$ to get a function

$$f: U \rightarrow \mathbb{C}_\infty.$$

The extended f is continuous.

In fact, $1/f$ is holomorphic at a pole p (defn. of “ $f: U \rightarrow \mathbb{C}_\infty$ is holomorphic at p ”).

Definition

A holomorphic function $f: U \setminus S \rightarrow \mathbb{C}$ with poles on a discrete set $S \subset U$ is said to be *meromorphic*.

If $p \in S$ is a pole, set $f(p) = \infty$ to get a function

$$f: U \rightarrow \mathbb{C}_\infty.$$

The extended f is continuous.

In fact, $1/f$ is holomorphic at a pole p (defn. of " $f: U \rightarrow \mathbb{C}_\infty$ is holomorphic at p ").

A *meromorphic function* is "a holomorphic function $f: U \rightarrow \mathbb{C}_\infty$."

Definition

A holomorphic function $f: U \setminus S \rightarrow \mathbb{C}$ with poles on a discrete set $S \subset U$ is said to be *meromorphic*.

If $p \in S$ is a pole, set $f(p) = \infty$ to get a function

$$f: U \rightarrow \mathbb{C}_\infty.$$

The extended f is continuous.

In fact, $1/f$ is holomorphic at a pole p (defn. of “ $f: U \rightarrow \mathbb{C}_\infty$ is holomorphic at p ”).

A *meromorphic function* is “a holomorphic function $f: U \rightarrow \mathbb{C}_\infty$.”

Technicality: Should we consider the constant ∞ a meromorphic function?

Definition

A holomorphic function $f: U \setminus S \rightarrow \mathbb{C}$ with poles on a discrete set $S \subset U$ is said to be *meromorphic*.

If $p \in S$ is a pole, set $f(p) = \infty$ to get a function

$$f: U \rightarrow \mathbb{C}_\infty.$$

The extended f is continuous.

In fact, $1/f$ is holomorphic at a pole p (defn. of “ $f: U \rightarrow \mathbb{C}_\infty$ is holomorphic at p ”).

A *meromorphic function* is “a holomorphic function $f: U \rightarrow \mathbb{C}_\infty$.”

Technicality: Should we consider the constant ∞ a meromorphic function?

In this course, we do not.

Definition

A holomorphic function $f: U \setminus S \rightarrow \mathbb{C}$ with poles on a discrete set $S \subset U$ is said to be *meromorphic*.

If $p \in S$ is a pole, set $f(p) = \infty$ to get a function

$$f: U \rightarrow \mathbb{C}_\infty.$$

The extended f is continuous.

In fact, $1/f$ is holomorphic at a pole p (defn. of “ $f: U \rightarrow \mathbb{C}_\infty$ is holomorphic at p ”).

A *meromorphic function* is “a holomorphic function $f: U \rightarrow \mathbb{C}_\infty$.”

Technicality: Should we consider the constant ∞ a meromorphic function?

In this course, we do not.

To emphasize we will often say “Let $f: U \rightarrow \mathbb{C}_\infty$ be meromorphic.”

One could define functions on $U \subset \mathbb{C}_\infty$ (like we did with LFTs).

One could define functions on $U \subset \mathbb{C}_\infty$ (like we did with LFTs).

If $\infty \in U$, $f: U \rightarrow \mathbb{C}_\infty$ is holomorphic at ∞ if $f(1/z)$ is holomorphic at 0.

One could define functions on $U \subset \mathbb{C}_\infty$ (like we did with LFTs).

If $\infty \in U$, $f: U \rightarrow \mathbb{C}_\infty$ is holomorphic at ∞ if $f(1/z)$ is holomorphic at 0.

So an LFT is a biholomorphic mapping $\mathbb{C}_\infty \rightarrow \mathbb{C}_\infty$.

One could define functions on $U \subset \mathbb{C}_\infty$ (like we did with LFTs).

If $\infty \in U$, $f: U \rightarrow \mathbb{C}_\infty$ is holomorphic at ∞ if $f(1/z)$ is holomorphic at 0.

So an LFT is a biholomorphic mapping $\mathbb{C}_\infty \rightarrow \mathbb{C}_\infty$.

Exercise: Show that a holomorphic $f: \mathbb{C}_\infty \rightarrow \mathbb{C}_\infty$ has at most finitely many poles and finitely many zeros.

One could define functions on $U \subset \mathbb{C}_\infty$ (like we did with LFTs).

If $\infty \in U$, $f: U \rightarrow \mathbb{C}_\infty$ is holomorphic at ∞ if $f(1/z)$ is holomorphic at 0.

So an LFT is a biholomorphic mapping $\mathbb{C}_\infty \rightarrow \mathbb{C}_\infty$.

Exercise: Show that a holomorphic $f: \mathbb{C}_\infty \rightarrow \mathbb{C}_\infty$ has at most finitely many poles and finitely many zeros.

Exercise: Show that a holomorphic $f: \mathbb{C}_\infty \rightarrow \mathbb{C}_\infty$ is either constant or onto.

One could define functions on $U \subset \mathbb{C}_\infty$ (like we did with LFTs).

If $\infty \in U$, $f: U \rightarrow \mathbb{C}_\infty$ is holomorphic at ∞ if $f(1/z)$ is holomorphic at 0.

So an LFT is a biholomorphic mapping $\mathbb{C}_\infty \rightarrow \mathbb{C}_\infty$.

Exercise: Show that a holomorphic $f: \mathbb{C}_\infty \rightarrow \mathbb{C}_\infty$ has at most finitely many poles and finitely many zeros.

Exercise: Show that a holomorphic $f: \mathbb{C}_\infty \rightarrow \mathbb{C}_\infty$ is either constant or onto.

Exercise: Show that a holomorphic $f: \mathbb{C}_\infty \rightarrow \mathbb{C}_\infty$ is a rational function (a polynomial divided by a polynomial).

One could define functions on $U \subset \mathbb{C}_\infty$ (like we did with LFTs).

If $\infty \in U$, $f: U \rightarrow \mathbb{C}_\infty$ is holomorphic at ∞ if $f(1/z)$ is holomorphic at 0.

So an LFT is a biholomorphic mapping $\mathbb{C}_\infty \rightarrow \mathbb{C}_\infty$.

Exercise: Show that a holomorphic $f: \mathbb{C}_\infty \rightarrow \mathbb{C}_\infty$ has at most finitely many poles and finitely many zeros.

Exercise: Show that a holomorphic $f: \mathbb{C}_\infty \rightarrow \mathbb{C}_\infty$ is either constant or onto.

Exercise: Show that a holomorphic $f: \mathbb{C}_\infty \rightarrow \mathbb{C}_\infty$ is a rational function (a polynomial divided by a polynomial).

Exercise: Show that an injective holomorphic $f: \mathbb{C}_\infty \rightarrow \mathbb{C}_\infty$ is an LFT.